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Possibility of Self-Focusing Due to Intensity Dependent Anomalous Dispersion

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Abstract

Close to an atomic resonance, the dependence of the real part of the susceptibility on power level (i.e. saturation effects) can be sizeable and therefore lead to self-trapping or self-focusing. It is shown here under what conditions this effect can occur. Order of magnitude estimates are given for the intensity dependent index change and the diameter of the trapped beam for some laser materials.

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I. Introduction

This is an analysis of the influence of intensity dependent anomalous dispersion on the propagation of electromagnetic radiation. The frequency of the electromagnetic field is assumed to lie in the vicinity of the resonance of an atomic transition and the nonlinear effects are assumed to arise from saturation of level population. In this paper particular attention is given to the propagation of laser radiation in a saturated amplifying medium. In a number of physical situations, the intensity dependent index of refraction due to this effect can be several times larger than the threshold index change necessary for the onset of self-trapping.

The details of intensity dependent anomalous dispersion and its influence on the extent of self-trapping differ appreciably from those encountered in liquids with large optical Kerr coefficients. Some considerations concerning the present effect are as follows:

1. Because of the assumption of near resonance the self-trapping effect considered here is highly frequency sensitive. In particular, in an amplifier self-trapping can only occur on the low frequency side of the resonance line.

2. In addition to the intensity dependence of the index of refraction, the intensity dependence of the gain coefficient of the resonance line also plays an important role in self-trapping. If the gain and index saturate at the same power level the focusing effect can be significantly reduced. There also exist important cases in which the intensity dependence of the refractive index is

appreciably different from that of the gain, and can thereby give rise to strong focusing. This can occur due to the presence of inhomogeneous broadening in a resonance line. Therefore, in the case of gaseous systems, Doppler and pressure effects determine the important parameters of self-trapping. For impurity resonances in solids, the impurity concentration and the operating temperature become of importance. These dependencies may be used to control the occurrence of self-trapping. Furthermore, study of the self-trapping due to intensity dependent anomalous dispersion may be used to obtain important spectroscopic information on the line shape of the resonance.

3. The nonlinear effect arising from saturation of a resonance line generally occurs at a power level lower than the onset of the nonlinearity due to the optical frequency Kerr effect. This enables one to use high gain laser transitions for the observation of self-trapping at the power levels obtainable from these lasers. However, the intensity dependence of a resonance line generally saturates rapidly as the power level increases. This effect is the determining factor in the final size of the trapped beam. In the case of the Kerr effect in liquids the corresponding effect, namely, the saturation of the intensity dependent index change occurs at a very much higher power level.

In Section II the self-focusing process is briefly described and the self-trapping condition and expected beam size are given. Section III discusses the intensity dependent index and gain for homogeneously and inhomogeneously broadened lines. Computer calculations of beam propagation are given for both types of broadening. Section IV points out some experimental possibilities.

II. The Self-Focusing Process

The details of the self-focusing effect due to ~~a nonlinear~~ an intensity dependent increase in index of refraction have been treated elsewhere.[1,2] We give, therefore, only a brief description of the self-focusing process. An increase in index of refraction with the intensity of a light beam produces a decrease in phase velocity in the regions where the beam is most intense. Thus, as the beam propagates the equiphase surface becomes more and more concave in the intense regions. From Huygen's principle, therefore, the rays should move toward the region of highest intensity and the intensity of the center should increase. This is, of course, just the opposite of what occurs in normal light diffraction. The distance in which the beam increases in intensity by a sizeable amount is given by

$$z_{focus} = \frac{d}{4}\left(\frac{n_o}{\delta n}\right)^{1/2} \tag{1}$$

where d is the diameter of the beam and δn is the intensity dependent increase in index of refraction. n_o is the normal index of refraction.

Self-focusing is possible when

$$\delta n \geqslant \delta n_{cr} \equiv \frac{1}{8n_o}\left(\frac{1.22\,\lambda}{d}\right)^2 \quad , \tag{2}$$

where λ is the wavelength. The equality implies that the condition for the lowest order trapped mode holds. In this case the focusing effect is exactly compensated for by diffraction. From the above we see that if δn reaches a maximum value, say through saturation, then there is a corresponding minimum beam diameter

$$d_{min} = 1.22\lambda(8n_o \ \delta n_{max})^{-1/2} \quad . \tag{3}$$

III. Theory of the Intensity Dependent Index and its Focusing Effect

We now consider in some detail the intensity dependence of the index due to an atomic or molecular transition as well as the intensity dependence of the gain from the same transition. The comparative dependence of the index and the gain on intensity is of crucial importance in determining the conditions under which self-focusing and self-trapping can occur.

If the frequency of the radiation field is considerably removed from the peak of the resonance line, saturation of dispersion and diffraction play the major roles in the propagation of the field. However, for a case in which the frequency of the field is at the center of the transition the dispersion effect is absent; the gain, including saturation, and the effect of diffraction determine the propagation of the field. The more interesting effect occurs when the frequency of the field is such that the influence of gain and nonlinear index both need to be considered. We confine our attention primarily to this region. In this connection, it is important to note the critical role of initial conditions.

A. Homogeneously Broadened Resonance

We discuss first the homogeneously broadened case, the rather more interesting inhomogeneous case can then be discussed with some modification. We consider first a two-level system in which we have the well-known result for the index change and gain

$$\delta n_{total}(\vec{r}) = \frac{2\pi}{n_o} \chi''_{res}(\omega_o)(\omega-\omega_o)T$$

$$\times \left\{1 + (\omega-\omega_o)^2 T^2 + \hbar^{-2}\wp^2 \, |E(\vec{r})|^2 T'T\right\}^{-1} \qquad (4a)$$

$$g(\vec{r}) = \frac{2(2\pi)^2}{\lambda n_o} \chi''_{res}(\omega_o)$$

$$\times \left\{1 + (\omega-\omega_o)^2 T^2 + \hbar^{-2}\wp^2 \, |E(\vec{r})|^2 T'T\right\}^{-1} \quad . \qquad (4b)$$

In the above, $\wp$ is the dipole matrix element, $|E(\vec{r})|^2$ is $2\pi/cn_o$ times the intensity, T is the inverse of the linewidth, ω is the operating frequency, and ω_o is the resonance frequency. Also, $\chi''_{res}(\omega_o)$ is the imaginary part of the peak field independent susceptibility. T′ is a characteristic time which is determined by the lifetimes of the atomic levels and is different for transient as opposed to steady state response. The subscript "total" on δn indicates that both the field independent and field dependent parts are included. δn_{total} is proportional to χ', the real part of the atomic or molecular susceptibility, and g is proportional to χ'', the imaginary part of the susceptibility. Considering the fact that the output of gas and other lasers often comes from a saturated transition, it is expected for these systems that $\hbar^{-2}\gamma^2 \, |E(\vec{r})|^2 \, T'T$ is comparable to or greater than unity. To find the intensity dependent ~~nonlinear~~ part of the index we subtract the intensity independent (E = 0) part from the above to obtain

$$\delta n(\vec{r}) = -\delta n_o \left\{ \left[\hbar^{-2}\gamma^2 |E(\vec{r})|^2 T'T \right] \left[1 + (\omega-\omega_o)^2 T^2 + \hbar^{-2}\gamma^2 |E(\vec{r})|^2 T'T \right]^{-1} \right\} \tag{5}$$

where

$$\delta n_o = \frac{2\pi}{n_o} \chi''_{res}(\omega_o)(\omega-\omega_o) T \left[1 + (\omega-\omega_o)^2 \, T^2 \right]^{-1} . \tag{6}$$

For an inverted system δn is negative on the low frequency side of the atomic or molecular resonance. It is evident that the factor in curly brackets in (5) approaches unity when ~~the non-linear index~~ δn saturates. For systems which are well saturated we have

$$\delta n \approx \delta n_{max} \equiv -\delta n_o \quad . \tag{7}$$

It is important to note in the above that both the gain and the index change saturate in the same way. This is not so for the inhomogeneous broadening case discussed below.

By merely modifying[3] the propagation equation of reference 2, employing equations (4b) and (5), we have studied, using a computer, the homogeneous broadening case in a traveling wave amplifier. A typical result for a cylindrical beam is shown in Fig. 1. Axial distance indicates propagation direction. In this case $(\omega_o-\omega)T = 1$. Here the axial distance is given in units of $d^2/4\lambda$. Also $|E|^2$ is plotted in units of the initial value at the beam center. The initial profile was Gaussian. It is evident that the rapid saturation of index has not allowed appreciable focusing to take place. This occurs because the gain and index profile flatten simultaneously. Note, however, the absence of diffraction effects. When $(\omega_o-\omega)T \gg 1$, $|\chi''| \ll |\chi'|$; hence the flattening influence of gain on the index profile will not be as large and it is expected that the focusing effect will be more pronounced.

B. Inhomogeneously Broadened Resonance

Next we consider the situation in which the levels are inhomogeneously broadened as, for example, by the molecular motion in a gas or by variations in crystal field in ruby. "Motional" narrowing due to collisions or some similar mechanism is assumed negligible.

In the case of a Doppler broadened transition the distribution of atomic frequencies is Gaussian. When the collision or natural width of the transition is comparable or less than the Doppler width the dependence of χ' and χ'' on field intensity cannot be expressed in closed form. The same is true for other types of inhomogenous broadening, for example, that which occurs in ruby. To avoid unnecessary complications we will assume the distribution of atomic frequencies is Lorentzian. While the Lorentzian distribution is only a crude approximation to most inhomogeneous distributions, nevertheless it shows the essential difference between the saturation properties of the gain and index. It has the great advantage that the result can be given a simple closed form. Convolving a Lorentzian of inhomogeneous width T_I^{-1} with the expressions for index and gain given by (4) we obtain

$$\delta n_{total}(\vec{r}) = \frac{2\pi}{n_o} \chi''_{res}(\omega_o)(\omega-\omega_o) T_I (T_I + T) T^{-1}$$
$$\times \left\{ (\omega-\omega_o)^2 T_I^2 + \left(1 + T_I T^{-1} \left[1 + \hbar^{-2} \wp^2 |E(\vec{r})|^2 T T'\right]^{1/2}\right)^2 \right\}^{-1} \tag{8a}$$

$$g(\vec{r}) = \frac{2(2\pi)^2}{\lambda n_o} \chi''_{res}(\omega_o) \ (T_I + T) \ T^{-1} \left[1 + \hbar^{-2} \wp^2 |E(\vec{r})|^2 T T'\right]^{-1/2}$$
$$\times \left\{1 + T_I T^{-1} \left[1 + \hbar^{-2} \wp^2 |E(\vec{r})|^2 T T'\right]^{+1/2}\right\}$$
$$\times \left\{ (\omega-\omega_o)^2 T_I^2 + \left(1 + T_I T^{-1} \left[1 + \hbar^{-2} \wp^2 |E(\vec{r})|^2 T T'\right]^{1/2}\right)^2 \right\}^{-1} . \tag{8b}$$

where $\chi''_{res}(\omega_o)$ is now the imaginary part of the peak field independent inhomogeneous susceptibility. For the inhomogeneous broadening to be significant $T_I T^{-1} \lesssim 1$. The important point to notice here is if $T_I T^{-1} << 1$ the gain saturates at much lower light intensity than the index. This is seen by the following conditions

$$\hbar^{-2} \wp^2 |E(\vec{r})|^2 T T' > 1 \qquad \text{Gain saturation}$$
$$\hbar^{-2} \wp^2 |E(\vec{r})|^2 T^{-1} T' T_I^2 > 1 \qquad \text{Index saturation} \quad . \tag{9}$$

Note that the relative change is the same for both the gain and index. What (9) indicates is that the change due to intensity occurs at lower intensities for the gain than for the index. This is made clear by Fig. 2 which shows the gain and index versus intensity for the homogeneous and inhomogeneous cases.

In Fig. 3 a computer solution is shown for the inhomogeneous broadening case under similar conditions to those used in the homogeneous case shown in Fig. 1, except that equations (8) were used after subtracting out the intensity independent index in (8a). The values $T_I T^{-1} = .1$ and $(\omega_o - \omega) T_I = 1$ were used. Because of the difference in saturation properties marked focusing has occurred. In this case the gain saturates quickly but the index profile remains, allowing the beam to focus. Note that this focusing is not as catastrophic as that found in reference 2 and it is expected that the beam will tend to the diameter given by equation (3). However, if $(\omega_o - \omega) T_I$ is of the order of 5 or 10 the intensity of the beam center as a function of axial distance shows a much more rapid focusing effect. It should be noted that in a gaseous system for the standing wave case the result is again similar to (4) and occurs because of the same effect which leads to the Lamb dip. In this case expressions (4) and (5) still roughly hold when $|\omega_o - \omega| T > 1$.

IV. Experimental Possibilities and Discussion

In addition to the requirement of inhomogeneous broadening it is necessary that the atomic distributions and the level populations be reasonably homogeneous over the cross-section of the beam so that spatial inhomogeneities in the intensity independent index and in the gain do not obscure the intensity dependent focusing effect. Kogelnick[4] has recently considered some of these intensity independent effects.

In Table I, values of gain, total intensity dependent index change δn_{max}, and the minimum beam diameter are given for some typical systems. The known values of gain are used to find $\chi''_{res}(\omega_o)$ and $\chi''_{res}(\omega_o)$ is used in turn to find δn_{max} assuming $(\omega_o - \omega)T_I = 1$. d_{min} is found from δn_{max} using (3). It should be noted that ruby and GaAs may also show focusing and trapping due to electrostriction.

It is interesting to compare the size of the index change due to ~~nonlinear~~ intensity dependent anomalous dispersion as given in Table I with that due to the Kerr effect in CS_2. In CS_2, $\delta n^{(2)}$ the quadratic index change is of the order 10^{-6} to 10^{-7} before the beam forms filaments and of the order of 10^{-2} to 10^{-3} in 4μ filaments.

The effect discussed might account for the filamentary nature of the output of Q-switched ruby lasers as well as in other laser systems such as semiconductor lasers. According to our analysis the presence of filamentary structure should be accompanied by operation of the laser on the low frequency side of the resonance. It should also be mentioned that important effects on the operating characteristics of the laser may occur because self-trapping will lead to propagation through the region where the density of excited atoms is maximum; in other words, diffraction losses can become unimportant.

It should be emphasized that the computer calculations shown in Figs. 1 and 3 are for the case of an amplifier. The trapping effect is strongly dependent on the initial conditions of intensity and beam shape. Therefore for an oscillator where feedback is important, additional considerations must be included in order to treat the details of trapping. Note that there are situations where the oscillator can be operated on a cavity mode such that $(\omega_o-\omega)T_I$ is comparable to or greater than unity. The same conditions may be achieved in an amplifier by tuning the input frequency.

Finally, note that we have discussed for simplicity only the case of saturation focusing due to operation on the low frequency side of an amplifying atomic line. Three other cases can be considered:

a) defocusing due to operation on the high frequency side of an amplifying transition,

b) focusing due to operation on the high frequency side of an attenuating transition,

c) defocusing due to operation on the low frequency side of an attenuating transition.

These three cases are also of great interest. Case b should be the most practical one for many experiments.

References

1. R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Letters 13, 479 (1964).

2. P. L. Kelley, Phys. Rev. Letters 15, 1005 (1965).

3. To obtain the equation used in the present calculation the replacement

$$\frac{\epsilon_2' k^2}{\epsilon_o} |E'| \rightarrow \frac{2k^2 \delta n(\vec{r})}{n_o} - ikg(\vec{r})$$

is made in equation (4) of reference 2.

4. H. Kogelnick, Appl. Opt. 4, 1562 (1965).

Table I. Nonlinear index changes and beam diameters expected in some laser materials. Note, $(\omega_o - \omega)T_I = 1$.

	Gain	δn_{max}	d_{min}
Xe	0.15/cm	$2. \times 10^{-6}$	1 mm
CO_2	0.014/cm	$6. \times 10^{-7}$	5 mm
Ruby	0.3/cm	$9. \times 10^{-7}$	.25 mm
GaAs	75/cm	$5. \times 10^{-3}$	4 μ

Figure Captions

Fig. 1 Growth of an Optical Beam for the Case of Homogeneous Line Broadening. The upper curve is the intensity of the beam center, the lower curve is the intensity at the 1/e distance from the center in the Gaussian initial beam profile.

Fig. 2 Nonlinear Index and Gain as a Function of Field Strength. The nonlinear index is given by the solid curves. The gain is given by the dashed curves. Both nonlinear index and gain are normalized to unity. $|E|$ is in units of $(\hbar^2 \gamma^2 TT')^{-1/2}$, also $T\,T_I^{-1} = 5$.

Fig. 3 Growth of an Optical Beam for the Case of Inhomogeneous Broadening. As in figure 1 the upper curve is the intensity of the beam center, the lower curve is the intensity at the 1/e distance from the beam center in the initial Gaussian profile.

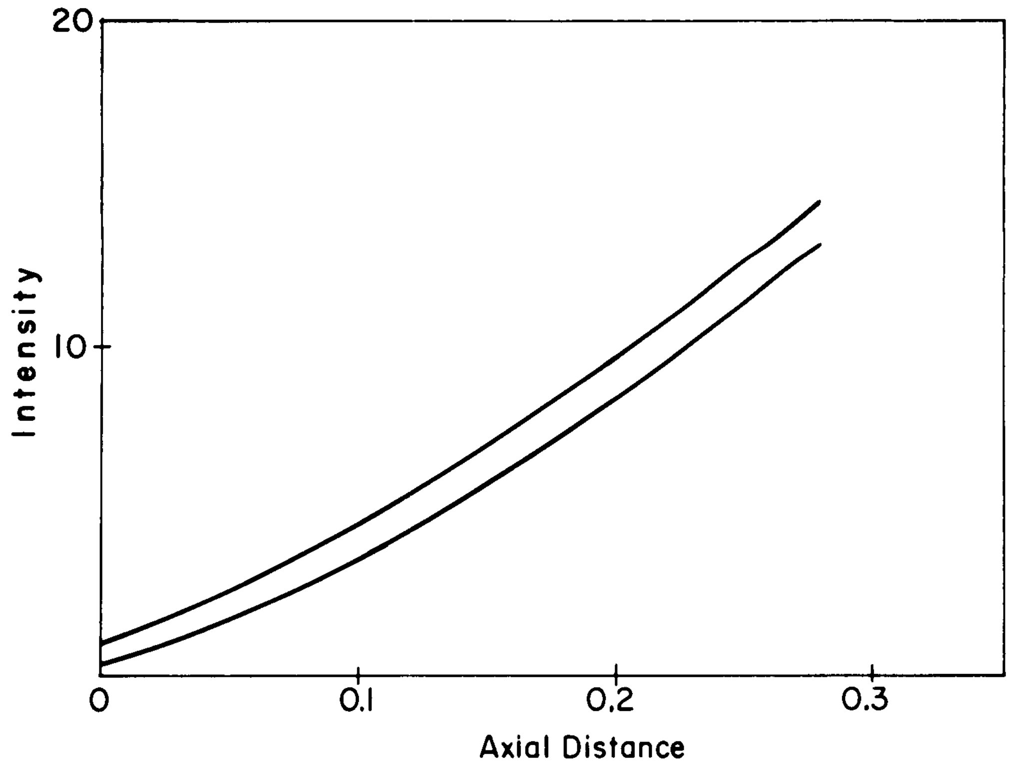

Fig. 1

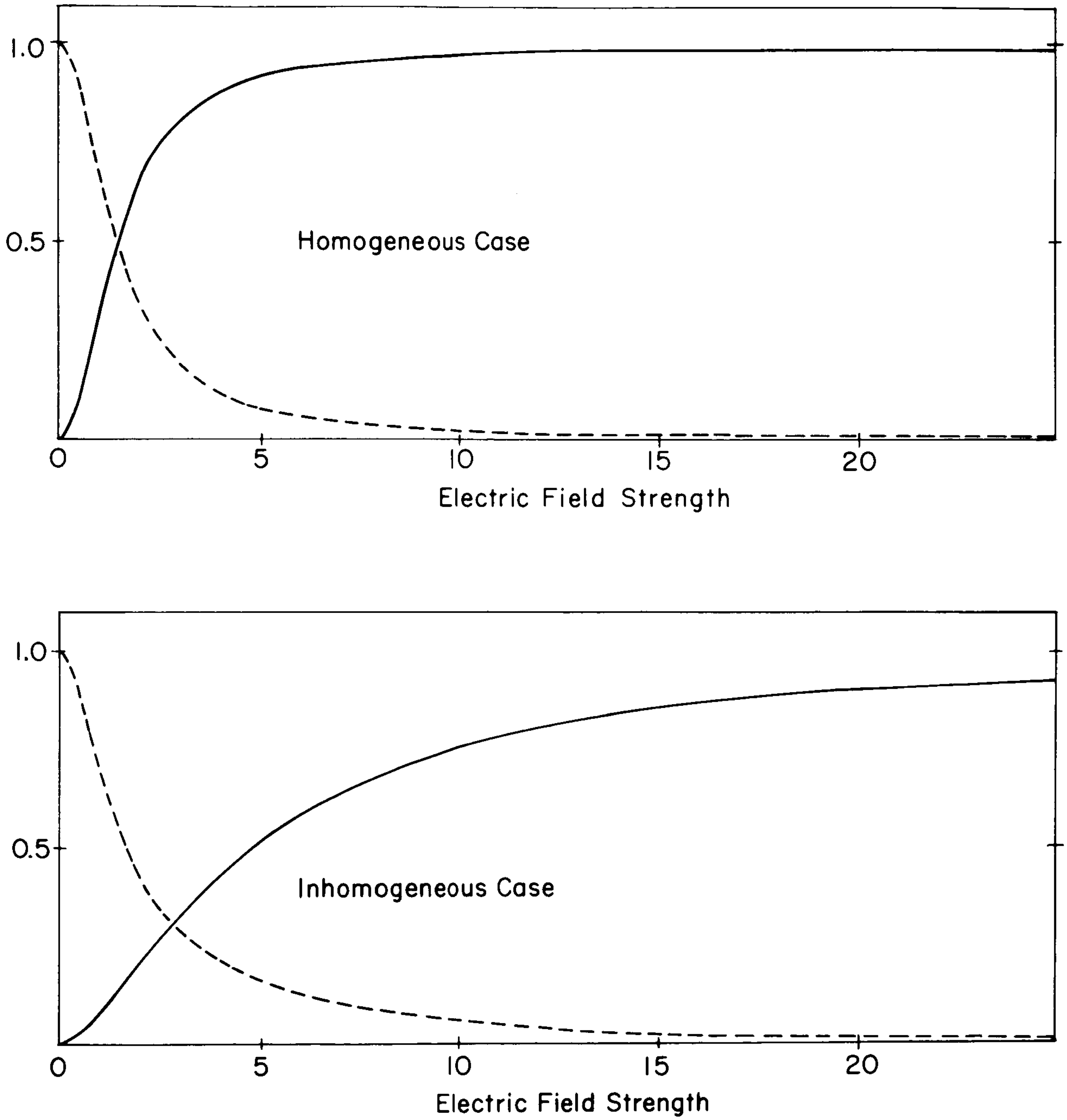
1.0
0.5
Homogeneous Case
0
5
10
15
20
Electric Field Strength
1.0
0.5
Inhomogeneous Case
0
5
10
15
20
Electric Field Strength

Fig. 2

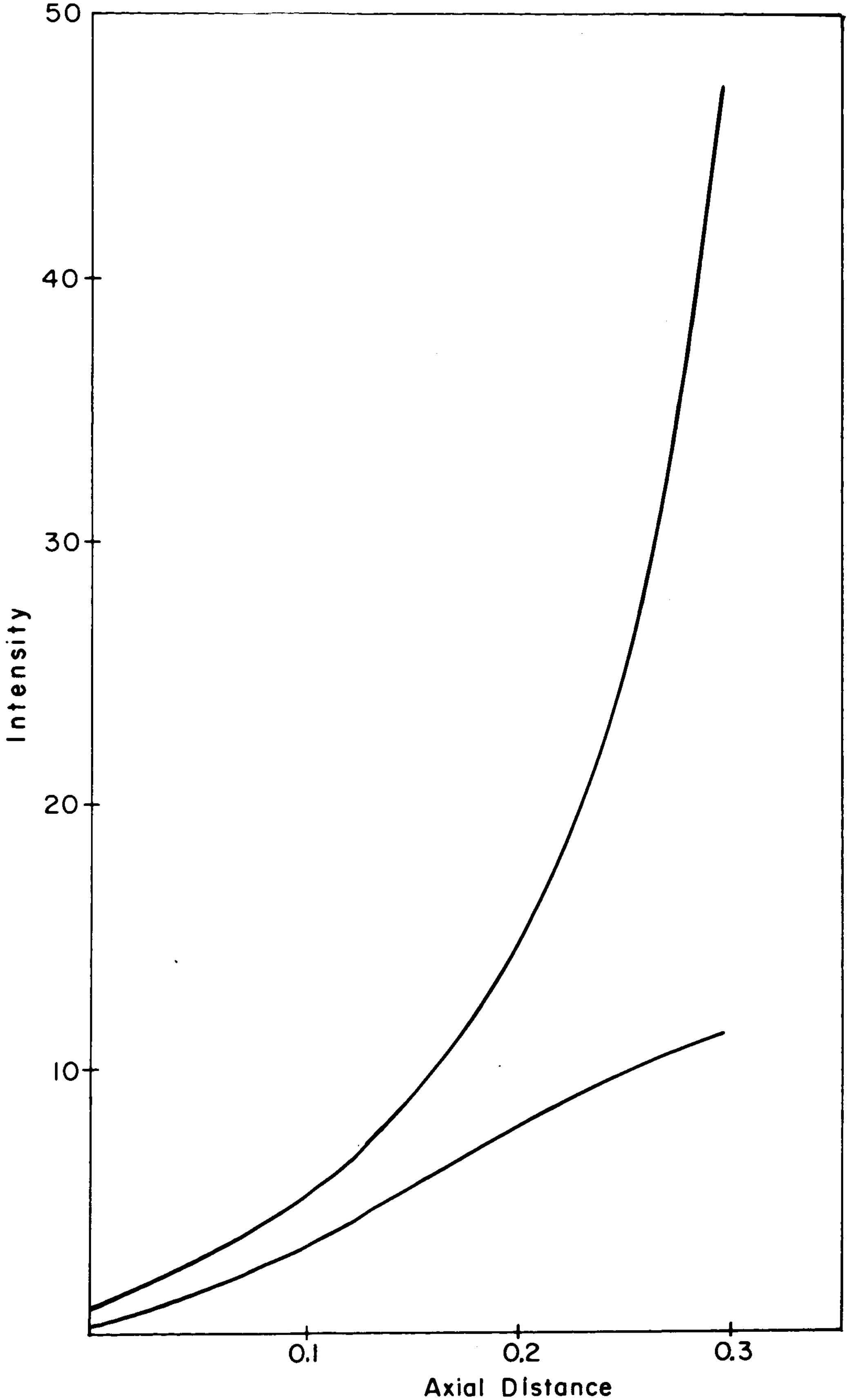

Fig. 3